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Asymmetric Wormholes via Electrically Charged Lightlike Branes

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Abstract

We consider a self-consistent Einstein-Maxwell-Kalb-Ramond system in the bulk $D = 4$ space-time interacting with a variable-tension electrically charged *lightlike* brane. The latter serves both as a material and charge source for gravity and electromagnetism, as well as it *dynamically* generates a bulk space varying cosmological constant. We find an *asymmetric wormhole* solution describing two "universes" with different spherically symmetric black-hole-type geometries connected through a "throat" occupied by the lightlike brane. The electrically neutral "left universe" comprises the exterior region of Schwarzschild-de-Sitter (or pure Schwarzschild) space-time above the *inner* (Schwarzschild-type) horizon, whereas the electrically charged "right universe" consists of the exterior Reissner-Nordström (or Reissner-Nordström-de-Sitter) black hole region beyond the *outer* Reissner-Nordström horizon. All physical parameters of the wormhole are uniquely determined by two free parameters – the electric charge and Kalb-Ramond coupling of the lightlike brane.

1 Introduction

Lightlike brane (*LL-branes* for short) play an important role in general relativity as they enter the description of various physically important cosmological and astrophysical phenomena such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events [1]; (ii) the "membrane paradigm" [2] of black hole physics; (iii) the thin-wall approach to domain walls coupled to gravity [3–5].

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More recently, *LL-branes* became significant also in the context of modern non-perturbative string theory, in particular, as the so called H-branes describing quantum horizons (black hole and cosmological) [6], as Penrose limits of baryonic D-branes [7], *etc* (see also Refs. [8]).

In the pioneering papers [3–5] *LL-branes* in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, *i.e.*, by introducing them without specifying the Lagrangian dynamics from which they may originate[1]. On the other hand, we have proposed in a series of recent papers [10–13] a new class of concise Lagrangian actions, providing a derivation from first principles of the *LL-brane* dynamics.

There are several characteristic features of *LL-branes* which drastically distinguish them from ordinary Nambu-Goto branes:

(i) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(ii) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu-Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless* p-branes (for a review, see Ref. [14]) which rather resemble a p-dimensional continuous distribution of massless point-particles.

(iii) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref. [4]).

(iv) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential "inflation/deflation" time behaviour [11] – an effect similar to the "mass inflation" effect around black hole horizons [15].

In a series of recent papers [12, 13, 16] we have explored the novel possibility of employing *LL-branes* as natural self-consistent gravitational sources for wormhole space-times, in other words, generating wormhole solutions in self-consistent bulk gravity-matter systems coupled to *LL-branes* through dynamically derived world-volume *LL-brane* stress energy tensors. For a review of wormhole space-times, see Refs. [17, 18].

The possibility of a "wormhole space-time" was first hinted at in the work of Einstein and Rosen [19], where they considered matching at the horizon of two identical copies of the exterior Schwarzschild space-time region (subsequently called *Einstein-Rosen "bridge"*). The original Einstein-Rosen "bridge" manifold appears as a particular case of the construction of spherically symmetric wormholes produced by *LL-branes* as gravitational sources (Refs. [13, 16]; see also Section 5 below). The main lesson here is that consistency of Einstein equations of motion yielding the original Einstein-Rosen "bridge" as well-defined solution necessarily requires the presence of *LL-brane* energy-momentum tensor

[1] In a more recent paper [9] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

as a source on the right hand side. Thus, the introduction of *LL-brane* coupling to gravity brings the original Einstein-Rosen construction in Ref. [19] to a consistent completion[2].

More complicated examples of spherically and axially symmetric wormholes with Reissner-Nordström and rotating cylindrical geometry, respectively, have also been presented in Refs. [12, 13]. Namely, two copies of the outer space-time region of a Reissner-Nordström or rotating cylindrical black hole, respectively, are matched via *LL-brane* along what used to be the outer horizon of the respective full black hole space-time manifold. In this way we obtain a wormhole solution which combines the features of the Einstein-Rosen "bridge" on the one hand (with wormhole throat at horizon), and the features of Misner-Wheeler wormholes [21], *i.e.*, exhibiting the so called "charge without charge" phenomenon[1], on the other hand.

In the present note the results of Refs. [12, 13] will be extended to the case of *asymmetric* wormholes, describing two "universes" with different spherically symmetric geometries of black hole type connected via a "throat" occupied by the pertinent gravitational source – an electrically charged *LL-brane*. As a result of the well-defined world-volume *LL-brane* dynamics coupled self-consistently to gravity and bulk space-time gauge fields, it creates a "left universe" with Schwarzschild-de-Sitter geometry where the cosmological constant is dynamically generated, and a "right universe" with Reissner-Nordström geometry with dynamically generated Coulomb field-strength. Similarly, the *LL-brane* can dynamically generate a non-zero cosmological constant in the "right universe", in which case it connects a purely Schwarzschild "left universe" with a Reissner-Nordström-de-Sitter "right universe".

The presentation of the material goes as follows. In Section 2 we briefly review the reparametrization-invariant world-volume Lagrangian formulation of *LL-branes* in both the Polyakov-type and Nambu-Goto-type forms. In Section 3 we briefly describe the main properties of *LL-brane* dynamics in spherically symmetric gravitational backgrounds stressing particularly on the "horizon straddling" phenomenon and the dynamical cosmological constant generation. Section 4 contains our principal result – the explicit construction of

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[2] Let us particularly emphasize that here and in what follows we consider the Einstein-Rosen "bridge" in its original formulation in Ref. [19] as a four-dimensional space-time manifold consisting of two copies of the exterior Schwarzschild space-time region matched along the horizon. On the other hand, the nomenclature of "Einstein-Rosen bridge" in several standard textbooks (*e.g.* Ref. [20]) uses the Kruskal-Szekeres manifold. The latter notion of "Einstein-Rosen bridge" is not equivalent to the original construction in Ref. [19]. Namely, the two regions in Kruskal-Szekeres space-time corresponding to the outer Schwarzschild space-time region ($r > 2m$) and labeled (I) and (III) in Refs. [20] are generally *disconnected* and share only a two-sphere (the angular part) as a common border ($U = 0, V = 0$ in Kruskal-Szekeres coordinates), whereas in the original Einstein-Rosen "bridge" construction the boundary between the two identical copies of the outer Schwarzschild space-time region ($r > 2m$) is a three-dimensional hypersurface ($r = 2m$).

[1] Misner and Wheeler [21] realized that wormholes connecting two asymptotically flat space times provide the possibility of "charge without charge", *i.e.*, electromagnetically non-trivial solutions where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

an asymmetric wormhole solution of self-consistent Einstein-Maxwell-Kalb-Ramond system interacting with an electrically charged *LL-brane*. The wormhole space-time consists of two "universes" with different spherically symmetric geometries connected through a "throat" occupied by the *LL-brane*: (a) electrically neutral "left universe" comprising the exterior region of Schwarzschild-de-Sitter (or pure Schwarzschild) space-time above the *inner* (Schwarzschild-type) horizon; (b) electrically charged "right universe" consisting of the exterior Reissner-Nordström (or Reissner-Nordström-de-Sitter) black hole region beyond the *outer* Reissner-Nordström horizon. In Section 5 we briefly consider the simple special case of the above construction with vanishing charge and Kalb-Ramond coupling of the *LL-branes*. It consistently describes the famous Einstein-Rosen "bridge" wormhole solution [19]. On the way we explain the crucial role of the presence of the *LL-brane* gravitational source producing the Einstein-Rosen "bridge" – an observation missing in the original classic paper [19].

2 Einstein-Maxwell-Kalb-Ramond System Interacting With Lightlike Brane: Lagrangian Formulation

Self-consistent bulk Einstein-Maxwell-Kalb-Ramond system coupled to a charged codimension-one *lightlike* p-brane (*i.e.*, $D = (p+1)+1$) is described by the following action:

$$S = \int d^D x \sqrt{-G} \left[\frac{R(G)}{16\pi} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{D!2}\mathcal{F}_{\mu_1\ldots\mu_D}\mathcal{F}^{\mu_1\ldots\mu_D}\right] + \widetilde{S}_{\rm LL} \; . \tag{1}$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ and

$$\mathcal{F}_{\mu_1\ldots\mu_D} = D\partial_{[\mu_1}\mathcal{A}_{\mu_2\ldots\mu_D]} = \mathcal{F}\sqrt{-G}\varepsilon_{\mu_1\ldots\mu_D} \tag{2}$$

are the field-strengths of the electromagnetic $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{D-1}}$ gauge potentials [22]. The last term on the r.h.s. of (1) indicates the reparametrization invariant world-volume action of the *LL-brane* coupled to the bulk gauge fields, proposed in our previous papers [10–12]:

$$\widetilde{S}_{\rm LL} = S_{\rm LL} - q\int d^{p+1}\sigma\, \varepsilon^{ab_1\ldots b_p} F_{b_1\ldots b_p}\partial_a X^\mu \mathcal{A}_\mu$$
$$-\frac{\beta}{(p+1)!}\int d^{p+1}\sigma\, \varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}\mathcal{A}_{\mu_1\ldots\mu_{p+1}} \; . \tag{3}$$

where:

$$S_{\rm LL} = \int d^{p+1}\sigma\, \Phi \left[-\frac{1}{2}\gamma^{ab}g_{ab} + L\big(F^2\big)\right] \; . \tag{4}$$

In Eqs.(3)–(4) the following notions and notations are used:

- Φ is alternative non-Riemannian integration measure density (volume form) on the p-brane world-volume manifold:

$$\Phi \equiv \frac{1}{(p+1)!}\varepsilon^{a_1\ldots a_{p+1}} H_{a_1\ldots a_{p+1}}(B) \quad , \tag{5}$$

$$H_{a_1\ldots a_{p+1}}(B) = (p+1)\partial_{[a_1} B_{a_2\ldots a_{p+1}]} \; , \tag{6}$$

instead of the usual $\sqrt{-\gamma}$. Here $\varepsilon^{a_1\ldots a_{p+1}}$ is the alternating symbol ($\varepsilon^{01\ldots p} = 1$), γ_{ab} ($a,b = 0,1,\ldots,p$) indicates the intrinsic Riemannian metric on the world-volume, and $\gamma = \det\|\gamma_{ab}\|$. $H_{a_1\ldots a_{p+1}}(B)$ denotes the field-strength of an auxiliary world-volume antisymmetric tensor gauge field $B_{a_1\ldots a_p}$ of rank p. As a special case one can build $H_{a_1\ldots a_{p+1}}$ (6) in terms of $p+1$ auxiliary world-volume scalar fields $\left\{\varphi^I\right\}_{I=1}^{p+1}$:

$$H_{a_1\ldots a_{p+1}} = \varepsilon_{I_1\ldots I_{p+1}} \partial_{a_1}\varphi^{I_1} \ldots \partial_{a_{p+1}}\varphi^{I_{p+1}} \; . \tag{7}$$

Note that γ_{ab} is *independent* of the auxiliary world-volume fields $B_{a_1\ldots a_p}$ or φ^I. The alternative non-Riemannian volume form (5) has been first introduced in the context of modified standard (non-lightlike) string and p-brane models in Refs. [23].

- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional space time with bulk Riemannian metric $G_{\mu\nu}(X)$ with $\mu,\nu = 0,1,\ldots,D-1$; $(\sigma) \equiv \left(\sigma^0 \equiv \tau, \sigma^i\right)$ with $i = 1,\ldots,p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.

- g_{ab} is the induced metric on world-volume:

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \; , \tag{8}$$

  which becomes *singular* on-shell (manifestation of the lightlike nature, cf. second Eq.(25) below).

- $L\left(F^2\right)$ is the Lagrangian density of another auxiliary $(p-1)$-rank antisymmetric tensor gauge field $A_{a_1\ldots a_{p-1}}$ on the world-volume with p-rank field-strength and its dual:

$$F_{a_1\ldots a_p} = p\partial_{[a_1} A_{a_2\ldots a_p]} \quad , \quad F^{*a} = \frac{1}{p!}\frac{\varepsilon^{aa_1\ldots a_p}}{\sqrt{-\gamma}} F_{a_1\ldots a_p} \; . \tag{9}$$

  $L\left(F^2\right)$ is *arbitrary* function of F^2 with the short-hand notation:

$$F^2 \equiv F_{a_1\ldots a_p} F_{b_1\ldots b_p} \gamma^{a_1 b_1} \ldots \gamma^{a_p b_p} \; . \tag{10}$$

Rewriting the action (4) in the following equivalent form:

$$S = -\int d^{p+1}\sigma\, \chi\sqrt{-\gamma}\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - L\big(F^2\big)\Big] \;\; , \; \chi \equiv \frac{\Phi}{\sqrt{-\gamma}} \tag{11}$$

with Φ the same as in (5), we find that the composite field χ plays the role of a *dynamical (variable) brane tension*. The notion of dynamical brane tension has previously appeared in different contexts in Refs. [24].

Let us also remark that, as it has been shown in Refs. [12, 28], the *LL-brane* equations of motion corresponding to the Polyakov-type action (4) (or (11)) can be equivalently obtained from the following *dual* Nambu-Goto-type action:

$$S_{\rm NG} = -\int d^{p+1}\sigma\, T \sqrt{\Bigl| \det \| g_{ab} - \epsilon \frac{1}{T^2}\partial_a u \partial_b u \| \Bigr|} \quad , \quad \epsilon = \pm 1 \; . \tag{12}$$

Here T is *dynamical* tension simply proportional to the dynamical tension in the Polyakov-type formulation (4) $(T \sim \chi = \frac{\Phi}{\sqrt{-\gamma}})$, and u denotes the dual potential w.r.t. $A_{a_1 \ldots a_{p-1}}$:

$$F^{*}_{a}(A) = \mathrm{const}\, \frac{1}{\chi} \partial_a u \; . \tag{13}$$

It what follows we will consider the original Polyakov-type action (4).

The pertinent Einstein-Maxwell-Kalb-Ramond equations of motion derived from the action (1) read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi \left(T^{(EM)}_{\mu\nu} + T^{(KR)}_{\mu\nu} + T^{(brane)}_{\mu\nu} \right) \; , \tag{14}$$

$$\partial_\nu \left(\sqrt{-G} \mathcal{F}^{\mu\nu} \right) + q \int d^{p+1}\sigma\, \delta^{(D)}\Bigl(x - X(\sigma) \Bigr) \varepsilon^{a b_1 \ldots b_p} F_{b_1 \ldots b_p} \partial_a X^\mu = 0 \; , \tag{15}$$

$$\varepsilon^{\nu\mu_1 \ldots \mu_{p+1}} \partial_\nu \mathcal{F} - \beta \int d^{p+1}\sigma\, \delta^{(D)}(x - X(\sigma)) \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} X^{\mu_1} \ldots \partial_{a_{p+1}} X^{\mu_{p+1}} = 0 \; , \tag{16}$$

where in the last equation we have used relation (2). The explicit form of the energy-momentum tensors read:

$$T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa} \mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa} \mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda} \; , \tag{17}$$

$$T^{(KR)}_{\mu\nu} = \frac{1}{(D-1)!} \left[\mathcal{F}_{\mu\lambda_1 \ldots \lambda_{D-1}} \mathcal{F}_{\nu}{}^{\lambda_1 \ldots \lambda_{D-1}} - \frac{1}{2D} G_{\mu\nu} \mathcal{F}_{\lambda_1 \ldots \lambda_D} \mathcal{F}^{\lambda_1 \ldots \lambda_D} \right] = -\frac{1}{2} \mathcal{F}^2 G_{\mu\nu} \tag{18}$$

$$T^{(brane)}_{\mu\nu} = - G_{\mu\kappa} G_{\nu\lambda} \int d^{p+1}\sigma \, \frac{\delta^{(D)}\Bigl(x - X(\sigma) \Bigr)}{\sqrt{-G}} \, \chi \, \sqrt{-\gamma} \gamma^{ab} \partial_a X^\kappa \partial_b X^\lambda \; , \tag{19}$$

where the brane stress-energy tensor is straightforwardly derived from the world-volume action (4) (or, equivalently, (11); recall $\chi \equiv \frac{\Phi}{\sqrt{-\gamma}}$ is the variable brane tension).

Eqs.(15)–(16) show that:

(i) the *LL-brane* is charged source for the bulk electromagnetism;

(ii) the *LL-brane* uniquely determines the value of $\mathcal{F}^2$ in Eq.(18) through its coupling to the bulk Kalb-Ramond gauge field (Eq.(16)) which implies *dynamical generation* of bulk cosmological constant $\Lambda = 4\pi \mathcal{F}^2$.

3 Lightlike Brane Dynamics in Spherically Symmetric Gravitational Backgrounds

The equations of motion of the *LL-brane* are discussed at length in our previous papers [10–13]. Their explicit form reads:

$$\partial_a \Big[\frac{1}{2}\gamma^{cd} g_{cd} - L(F^2)\Big] = 0 \quad \longrightarrow \quad \frac{1}{2}\gamma^{cd} g_{cd} - L(F^2) = M \; , \tag{20}$$

where M is an arbitrary integration constant;

$$\frac{1}{2} g_{ab} - F^2 L'(F^2) \left[\gamma_{ab} - \frac{F^*_a F^*_b}{F^{*\,2}}\right] = 0 \; , \tag{21}$$

where $F^{*\,a}$ is the dual world-volume field strength (9);

$$\partial_{[a}\left(F^*_{b]}\,\chi L'(F^2)\right) + \frac{q}{4}\partial_a X^\mu \partial_b X^\nu \mathcal{F}_{\mu\nu} = 0 \; ; \tag{22}$$

$$\begin{aligned} &\partial_a\left(\chi\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} \\ &\qquad - q\varepsilon^{ab_1\ldots b_p} F_{b_1\ldots b_p}\partial_a X^\nu \mathcal{F}_{\lambda\nu} G^{\lambda\mu} \\ &- \frac{\beta}{(p+1)!}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1} X^{\mu_1}\ldots\partial_{a_{p+1}} X^{\mu_{p+1}} \mathcal{F}_{\lambda\mu_1\ldots\mu_{p+1}} G^{\lambda\mu} = 0 \; . \end{aligned} \tag{23}$$

Here χ is the dynamical brane tension as in (11), $\mathcal{F}_{\lambda\mu_1\ldots\mu_{p+1}}$ is the Kalb-Ramond field-strength (2),

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right) \tag{24}$$

is the Christoffel connection for the external metric, and $L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2.

Eqs.(20)–(21) imply the following important consequences:

$$F^2 = F^2(M) = \mathrm{const} \quad , \quad g_{ab}F^{*\,b} = 0 \; , \tag{25}$$

where the second equation is the manifestation of the lightlike nature of the p-brane model (4), *i.e.*, the tangent vector to the world-volume $F^{*\,a}\partial_a X^\mu$ is *lightlike* w.r.t. metric of the embedding space-time.

World-volume reparametrization invariance allows us to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \;\; (i = 1,\ldots,p) \; , \; \gamma^{00} = -1 \; . \tag{26}$$

Also, we will use a natural ansatz for the "electric" part of the auxiliary world-volume gauge field-strength (9):

$$F^{*i} = 0 \;\; (i = 1,\ldots,p) \quad , \quad \text{i.e.} \;\; F_{0 i_1\ldots i_{p-1}} = 0 \; , \tag{27}$$

meaning that we choose the lightlike direction in Eq.(25) to coincide with the brane proper-time direction on the world-volume ($F^{*\,a}\partial_a \sim \partial_\tau$). The Bianchi identity ($\nabla_a F^{*a} = 0$) together with (26)–(27) and the definition for the dual field-strength in (9) imply:

$$\partial_0 \gamma^{(p)} = 0 \quad \text{where} \;\; \gamma^{(p)} \equiv \det \|\gamma_{ij}\| \; . \tag{28}$$

Taking into account (26)–(27), Eqs.(21) acquire the following gauge-fixed form (recall definition of the induced metric g_{ab} (8)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0 \quad , \quad g_{0i} = 0 \quad , \quad g_{ij} - 2a_0 \, \gamma_{ij} = 0 \; , \tag{29}$$

where a_0 is a M-dependent constant:

$$a_0 \equiv F^2 L'(F^2) \Big|_{F^2 = F^2(M)} \; . \tag{30}$$

Eqs.(29) are analogs of the Virasoro constraints in standard string theory.

In what follows we will be interested in static spherically symmetric solutions of Einstein-Maxwell-Kalb-Ramond equations (14)–(16). The generic form of spherically symmetric metric in Eddington-Finkelstein coordinates [25] reads:

$$ds^2 = -A(r) dv^2 + 2 dv\, dr + C(r) h_{ij}(\theta) d\theta^i d\theta^j \; , \tag{31}$$

where h_{ij} indicates the standard metric on S^p. We will consider the simplest ansatz for the *LL-brane* embedding coordinates:

$$X^0 \equiv v = \tau \quad , \quad X^1 \equiv r = r(\tau) \quad , \quad X^i \equiv \theta^i = \sigma^i \;\; (i = 1, \ldots, p) \tag{32}$$

Now, the *LL-brane* equations (29) together with (28) yield:

$$-A(r) + 2\, \dot{r} = 0 \quad , \quad \partial_\tau C = \dot{r}\; \partial_r C \Big|_{r = r(\tau)} = 0 \; , \tag{33}$$

implying:

$$\dot{r} = 0 \;\rightarrow\; r = r_0 = \text{const} \quad , \quad A(r_0) = 0 \; . \tag{34}$$

Eq.(34) tells us that consistency of *LL-brane* dynamics in a spherically symmetric gravitational background of codimension one requires the latter to possess a horizon (at some $r = r_0$), which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref. [4]). Similar property – "horizon straddling", has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [12, 13].

Next, the Maxwell coupling of the *LL-brane* produces via Eq.(15) static Coulomb field in the outer region beyond the horizon (for $r > r_0$). Namely, inserting in Eq.(15) the embedding ansatz (32) together with (34) and accounting for (26)–(29) we obtain;

$$\partial_r \left(C^{p/2}(r) \mathcal{F}_{vr}(r) \right) - q \, \frac{\sqrt{p! F^2}}{(2a_0)^{p/2}} C^{p/2}(r_0) \delta(r - r_0) = 0 \; , \tag{35}$$

which yields for the Maxwell field-strength:

$$\mathcal{F}_{vr}(r) = \left(\frac{C(r_0)}{C(r)} \right)^{p/2} \frac{q \sqrt{p! F^2}}{(2a_0)^{p/2}} \theta(r - r_0) \; . \tag{36}$$

Using again the embedding ansatz (32) together with (34) as well as (26)–(29), the Kalb-Ramond equations of motion (16) reduce to:

$$\partial_r \mathcal{F} + \beta\delta(r-r_0) = 0 \quad \to \quad \mathcal{F} = \mathcal{F}_{(+)}\theta(r-r_0) + \mathcal{F}_{(-)}\theta(r_0-r) \tag{37}$$

$$\mathcal{F}_{(\pm)} = \mathrm{const} \quad , \quad \mathcal{F}_{(-)} - \mathcal{F}_{(+)} = \beta \tag{38}$$

Therefore, a space-time varying non-negative cosmological constant is dynamically generated in both exterior and interior regions w.r.t. the horizon at $r = r_0$ (cf. Eq.(18)):

$$\Lambda_{(\pm)} = 4\pi\mathcal{F}^2_{(\pm)} \; . \tag{39}$$

Finally, it remains to consider the second order (w.r.t. proper time derivative) X^μ equations of motion (23). Upon inserting the embedding ansatz (32) together with (34) and taking into account (29), (36) and (38), we find that the only non-trivial equations is for $\mu = v$. Before proceeding let us note that the "force" terms in the X^μ equations of motion (23) (the geodesic ones containing the Christoffel connection coefficients as well as those coming from the *LL-brane* coupling to the bulk Maxwell and Kalb-Ramond gauge fields) contain discontinuities across the horizon occupied by the *LL-brane*. The discontinuity problem is resolved following the approach in Ref. [3] (see also the regularization approach in Ref. [26], Appendix A) by taking mean values of the "force" terms across the discontinuity at $r = r_0$. Thus, we obtain from Eq.(23) with $\mu = v$:

$$\partial_\tau \chi + \chi \left[\frac{1}{4}\left(\partial_r A_{(+)} + \partial_r A_{(-)}\right) + \frac{1}{2} p a_0 \left(\partial_r \ln C_{(+)} + \partial_r \ln C_{(-)}\right)\right]_{r=r_0} + \frac{1}{2}\Bigl[-q^2 \frac{p! F^2}{(2a_0)^{p/2}} + \beta (2a_0)^{p/2}\left(\mathcal{F}_{(-)} + \mathcal{F}_{(+)}\right)\Bigr] = 0 \; . \tag{40}$$

4 Asymmetric Wormhole Solution

The *LL-brane* energy-momentum tensor (19) on the r.h.s. of the Einstein equations of motion (14), upon inserting the expressions for $X^\mu(\sigma)$ from (32) and (34), and taking into account the gauge-fixing conditions (26) and the ansatz (27), acquires the form:

$$T^{\mu\nu}_{(brane)} = S^{\mu\nu}\,\delta(r-r_0) \tag{41}$$

with surface energy-momentum tensor:

$$S^{\mu\nu} \equiv \frac{\chi}{(2a_0)^{p/2}} \left[\partial_\tau X^\mu \partial_\tau X^\nu - 2a_0 G^{ij}\partial_i X^\mu \partial_j X^\nu\right]_{v=\tau,\, r=r_0,\, \theta^i=\sigma^i} \; . \tag{42}$$

Here a_0 is the integration constant parameter appearing in the *LL-brane* dynamics (30) and $G_{ij} = C(r)h_{ij}(\theta)$. For the non-zero components of $S_{\mu\nu}$ (with lower indices) and its trace we find:

$$S_{rr} = \frac{\chi}{(2a_0)^{p/2}} \quad , \quad S_{ij} = -\frac{\chi}{(2a_0)^{p/2-1}} G_{ij} \quad , \quad S^\lambda_\lambda = -\frac{p\chi}{(2a_0)^{p/2-1}} \tag{43}$$

The solution of the other bulk space-time equations of motion (the Maxwell (15) and Kalb-Ramond (16)) with spherically symmetric geometry have already been given in the previous Section, see Eqs.(35)–(38).

For the sake of simplicity we will consider in what follows the case of $D = 4$-dimensional bulk space-time and, correspondingly, $p = 2$ for the *LL-brane*. The generalization to arbitrary D is straightforward. For further simplification of the numerical constant factors we will choose the following specific ("wrong-sign" Maxwell) form for the Lagrangian of the auxiliary non-dynamical world-volume gauge field (cf. Eqs.(9)–(10)):

$$L(F^2) = \frac{1}{4}F^2 \quad \to \quad a_0 = M \; , \tag{44}$$

where again a_0 is the constant defined in (30) and M denotes the original integration constant in Eqs.(20).

We will show that there exists an *asymmetric wormhole* solution of the Einstein equations of motion (14) with *LL-brane* energy-momentum tensor on the r.h.s. given by (42)–(43) – systematically derived from the reparametrization invariant *LL-brane* world-volume action (4), which describes two "universes" with different spherically symmetric geometries (31) matched along a *LL-brane*. More specifically, this solution describes an overall space-time manifold containing two separate spherically symmetric space-time regions:

(i) a "left universe" consisting of the exterior region of Schwarzschild-de-Sitter space-time above the *inner* (Schwarzschild-type) horizon, *i.e.*, with metric (31) where $C(r) = r^2$ and:

$$A(r) \equiv A_{(-)}(r) = 1 - \frac{2m_1}{r} - Kr^2 \qquad \text{for } r > r_0 \; , \tag{45}$$

$$A_{(-)}(r_0) = 0 \quad , \quad \partial_r A_{(-)}\big|_{r=r_0} > 0 \; ; \tag{46}$$

(ii) a "right universe" comprising the exterior Reissner-Nordström black hole region beyond the *outer* Reissner-Nordström horizon with metric (31) where $C(r) = r^2$ and:

$$A(r) \equiv A_{(+)}(r) = 1 - \frac{2m_2}{r} + \frac{Q^2}{r^2} \qquad \text{for } r > r_0 \; , \tag{47}$$

$$A_{(+)}(r_0) = 0 \quad , \quad \partial_r A_{(+)}\big|_{r=r_0} > 0 \; . \tag{48}$$

The "throat" connecting the above two "universes" with metrics (45) and (47) is the lightlike world-volume hypersurface of the *LL-brane* which is located on the common horizon ($r = r_0$) of both "universes" – a Schwarzschild-type horizon from the "left universe" side (46) and an outer Reissner-Nordström horizon from the "right universe" side (48). As already pointed out above (Eqs.(33)–(34)), the common horizon at $r = r_0$ is automatically occupied by the *LL-brane* ("horizon straddling") as a result of its world-volume dynamics.

The asymmetric wormhole solution under consideration is explicitly given in Eddington-Finkelstein-type coordinates as:

$$ds^2 = -\widetilde{A}(\eta)dv^2 + 2dv\, d\eta + \widetilde{r}^2(\eta) \left[d\theta^2 + \sin^2\theta d\varphi^2\right] \; , \tag{49}$$

where the original radial coordinate r ($r > 0$) is replaced by η ($-\infty < \eta < \infty$) upon substituting:

$$r \quad \to \quad \widetilde{r}(\eta) = r_0 + |\eta| \tag{50}$$

with r_0 – the common horizon of (45) and (47) as follows:

$$\widetilde{A}(\eta) \equiv A_{(-)}\big(\widetilde{r}(\eta)\big) = 1 - \frac{2m_1}{\widetilde{r}(\eta)} - K\widetilde{r}^2(\eta) \quad , \quad K \equiv \frac{4\pi}{3}\beta^2 \;\; , \;\; \text{for } \eta < 0 \; , \tag{51}$$

$$\widetilde{A}(\eta) \equiv A_{(+)}\big(\widetilde{r}(\eta)\big) = 1 - \frac{2m_2}{\widetilde{r}(\eta)} + \frac{Q^2}{\widetilde{r}^2(\eta)} \quad , \quad Q^2 \equiv \frac{8\pi}{a_0} q^2\, r_0^4 \;\; , \;\; \text{for } \eta > 0 \; . \tag{52}$$

The new radial-like coordinate η describes a continuous interpolation between the left ($\eta < 0$) and the right ($\eta > 0$) “universes” through the “throat” at $\eta = 0$. As shown in Eqs.(37)–(39), the *LL-brane* through its coupling to the bulk Kalb-Ramond field (cf. (3) and (16)) dynamically generates space-time varying non-negative cosmological constant with a jump across the horizon ($r = r_0$). In the present case this yields dynamically generated de Sitter parameter $K = \frac{1}{3}\Lambda_{(-)} \equiv \frac{4\pi}{3}\mathcal{F}^2_{(-)} = \frac{4\pi}{3}\beta^2$ in the “left universe” as given in (51), whereas $\mathcal{F}_{(+)} = 0$. On the other hand, the surface charge density q of the *LL-brane* (cf. (15) and (35)) explicitly determines the non-zero Coulomb field-strength in the “right universe” with Q^2 as given in (52).

Now, substituting the metric (49) with (51)–(52) into the Einstein equations (14) and taking into account that outside the “throat” ($\eta = 0$) it obviously solves the “vacuum” equations (with the *LL-brane* absent), the only non-trivial δ-function contribution on the l.h.s. of (14) arises because of non-smoothness of the metric (49) with (51)–(52) at $\eta = 0$ (it is continuous but not differentiable there). Thus, inserting the world-volume Lagrangian-derived expression (41)–(42) for the *LL-brane* stress-energy tensor on the r.h.s. of Einstein Eqs.(14) yields (for $D = 4, p = 2$) two relations matching the coefficients in front of $\delta(\eta)$:

$$\Big[\partial_\eta \widetilde{A}_{(+)} - \partial_\eta \widetilde{A}_{(-)}\Big]_{\eta=0} = -16\pi\chi \quad , \quad r_0 = -\frac{a_0}{\pi\chi} \; , \tag{53}$$

The second relation (53) implies that the dynamical *LL-brane* tension χ must be constant (independent of the *LL-brane* proper time τ) and it must be *negative*. Substituting the explicit form (51)–(52) of $\widetilde{A}_\pm$ in (53) gives the following expressions for the mass parameters:

$$m_1 = \frac{a_0}{2\pi|\chi|}\Big(1 - \frac{4a_0^2\beta^2}{3\pi\chi^2}\Big) \quad , \quad m_2 = \frac{a_0}{2\pi|\chi|}\Big(1 + \frac{8a_0 q^2}{\pi\chi^2}\Big) \; , \tag{54}$$

as well as the relation between a_0 and χ:

$$\chi^2 = \frac{2a_0\,(2q^2 + a_0\beta^2)}{\pi(1 - 8a_0)} \; . \tag{55}$$

Next, the *LL-brane* equation of motion (40), where we set $\chi = \mathrm{const}$, yields in the case under consideration a third matching relation at the “throat”:

$$\frac{|\chi|}{4}\,\Big[\partial_\eta \widetilde{A}_{(+)} + \partial_\eta \widetilde{A}_{(-)}\Big]_{\eta=0} + 2q^2 - a_0\beta^2 = 0 \; , \tag{56}$$

which upon using (54)–(55) reduces to the second relation (53), *i.e.*, Eq.(56) does not carry any new information.

From relations (54)–(55) we conclude that all physical parameters of the asymmetric wormhole (51)–(52) are explicitly determined by the two free parameters (q, β) – the surface electric charge density q and the Kalb-Ramond charge β of the *LL-brane*.

It remains to check that:

$$\partial_\eta \widetilde{A}\big|_{\eta\to+0} \equiv \partial_r A_{(+)}\big|_{r=r_0} > 0 \quad , \quad -\partial_\eta \widetilde{A}\big|_{\eta\to-0} \equiv \partial_r A_{(-)}\big|_{r=r_0} > 0 \; , \tag{57}$$

i.e., the "throat" at $\eta = 0$ must be the outer Reissner-Nordström horizon from the point of view of the "right" Reissner-Nordström "universe" ($\eta > 0$, cf. (48)) and simultaneously be the inner Schwarzschild-type horizon from the point of view of the "left" Schwarzschild-de-Sitter "universe"($\eta < 0$, cf. (46)). From (54)–(55) we find:

$$\partial_\eta \widetilde{A}\big|_{\eta\to+0} = \frac{\pi\,|\chi|}{a_0(2q^2 + a_0\beta^2)} \left[2q^2(16a_0 - 1) + a_0\beta^2\right] > 0 \tag{58}$$

$$-\partial_\eta \widetilde{A}\big|_{\eta\to-0} = \frac{\pi\,|\chi|}{a_0(2q^2 + a_0\beta^2)} \left[2q^2 + a_0\beta^2(16a_0 - 1)\right] > 0 \; . \tag{59}$$

Inequalities (58)–(59) together with Eq.(55) imply the following restriction on the integration constant a_0 from the *LL-brane* dynamics (30):

$$1/16 < a_0 < 1/8 \; . \tag{60}$$

In complete analogy one can construct another asymmetric wormhole solution where the *LL-brane* connects the "left" universe, which is now the exterior region of the standard Schwarzschild space-time ($r > r_0 = 2m_1$), with the "right" universe, which is the exterior region of the Reissner-Nordström-de-Sitter space-time ($r > r_0$) beyond the outer Reissner-Nordström horizon $r = r_0$:

$$\widetilde{A}(\eta) \equiv A_{(-)}\big(\widetilde{r}(\eta)\big) = 1 - \frac{2m_1}{\widetilde{r}(\eta)} \quad \text{for } \eta < 0 \; , \tag{61}$$

$$\widetilde{r}(\eta) = r_0 - \eta \quad , \quad r_0 = 2m_1 \quad ,$$

$$\widetilde{A}(\eta) \equiv A_{(+)}\big(\widetilde{r}(\eta)\big) = 1 - \frac{2m_2}{\widetilde{r}(\eta)} + \frac{Q^2}{\widetilde{r}^2(\eta)} - K\widetilde{r}^2(\eta) \quad \text{for } \eta > 0 \; , \tag{62}$$

$$\widetilde{r}(\eta) = r_0 + \eta \quad , \quad Q^2 \equiv \frac{8\pi}{a_0} q^2\, r_0^4 \quad , \quad K \equiv \frac{4\pi}{3}\beta^2 \; .$$

In this case the physical parameters of the asymmetric wormhole read:

$$m_1 = \frac{r_0}{2} = \frac{a_0}{2\pi\chi} \quad , \quad m_2 = \frac{a_0}{2\pi\chi}\left[1 + \frac{2(1-8a_0)}{2q^2 + a_0\beta^2}\left(2q^2 - \frac{1}{3}a_0\beta^2\right)\right] \; , \tag{63}$$

for χ^2 we get the same relation (55) and the same restriction (60) holds for the *LL-brane* integration constant a_0.

5 A Note on Einstein-Rosen "Bridge"

In the simple special case $(q = 0, \beta = 0)$ the *asymmetric* wormhole solution (49)–(52) reduces to a *symmetric* wormhole solution:

$$ds^2 = -\widetilde{A}(\eta)dv^2 + 2dv\,d\eta + \widetilde{r}^2(\eta)\left[d\theta^2 + \sin^2\theta d\varphi^2\right] ,$$
$$\widetilde{A}(\eta) = 1 - \frac{2m}{\widetilde{r}(\eta)} \quad , \quad \widetilde{r}(\eta) = 2m + |\eta| . \tag{64}$$

The above metric describes two identical copies of Schwarzschild *exterior* space-time region ($r > 2m$), which correspond to $\eta > 0$ and $\eta < 0$, respectively, and which are "glued" together at the horizon $\eta = 0$ (*i.e.*, $r = 2m$) occupied by the *LL-brane*, where the latter serves as a throat of the overall wormhole solution. This is precisely the space-time manifold of the Einstein-Rosen "bridge" solution in its original formulation [19] in terms of Eddington-Finkelstein coordinates. An important consequence of the present construction is that the Einstein-Rosen "bridge" wormhole *does not* satisfy the vacuum Einstein equations since it needs the presence of a non-trivial matter stress-energy tensor on the r.h.s. (14) which turns out to be the stress-energy tensor of a *LL-brane* (41)–(42) self-consistently derived from a well-defined reparametrization-invariant world-volume *LL-brane* Lagrangian (4).

To make the connection with the original formulation [19] of the Einstein-Rosen "bridge" more explicit let us recall that Einstein and Rosen start from the standard Schwarzschild metric:

$$ds^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2\left[d\theta^2 + \sin^2\theta\, d\varphi^2\right] \quad , \quad A(r) = 1 - \frac{2m}{r} \tag{65}$$

and introduce new radial-like coordinate u by defining $u^2 = r - 2m$, so that the metric (65) becomes:

$$ds^2 = -\frac{u^2}{u^2 + 2m}dt^2 + 4(u^2 + 2m)du^2 + (u^2 + 2m)^2\left[d\theta^2 + \sin^2\theta\, d\varphi^2\right] . \tag{66}$$

Then Einstein and Rosen take two identical copies of the exterior Schwarzschild space-time region ($r > 2m$) by letting the new coordinate u to vary between $-\infty$ and $+\infty$ (*i.e.*, we have the same $r \geq 2m$ for $\pm u$). The two Schwarzschild exterior space-time regions must be matched at the horizon $u = 0$ (the wormhole "throat").

Let us examine whether the original Einstein-Rosen solution satisfy the vacuum Einstein equations everywhere. To this end let us consider the Levi-Civita identity (see *e.g.* [27]):

$$R^0_0 = -\frac{1}{\sqrt{-g_{00}}}\nabla^2\left(\sqrt{-g_{00}}\right) \tag{67}$$

valid for any metric of the form $ds^2 = g_{00}(r)(dt)^2 + h_{ij}(r,\theta,\varphi)dx^i dx^j$ and where ∇^2 is the three-dimensional Laplace-Beltrami operator $\nabla^2 =$

$\frac{1}{\sqrt{h}}\frac{\partial}{\partial x^i}\left(\sqrt{h}\,h^{ij}\frac{\partial}{\partial x^j}\right)$. The Einstein-Rosen metric (66) solves $R^0_0 = 0$ for all $u \neq 0$. However, since $\sqrt{-g_{00}} \sim |u|$ as $u \to 0$ and since $\frac{\partial^2}{\partial u^2}|u| = 2\delta(u)$, Eq.(67) tells us that:

$$R^0_0 \sim \frac{1}{|u|}\delta(u) \sim \delta(u^2) \; , \tag{68}$$

and similarly for the scalar curvature $R \sim \frac{1}{|u|}\delta(u) \sim \delta(u^2)$. From (68) we conclude that:

(i) The non-vanishing r.h.s. of (68) exhibits the explicit presence of some lightlike matter source on the throat – an observation which is missing in the original formulation [19] of the Einstein-Rosen "bridge". In fact, the problem with the metric (66) satisfying the vacuum Einstein equations at $u = 0$ has been noticed in ref. [19], where in Eq.(3a) the authors multiply Ricci tensor by an appropriate power of the determinant g of the metric (66) vanishing at $u = 0$ so as to enforce fulfillment of the vacuum Einstein equations everywhere, including at $u = 0$.

(ii) The coordinate u in (66) is *inadequate* for description of the original Einstein-Rosen "bridge" at the throat due to the *ill-definiteness* as distribution of the r.h.s. in (68).

As we have seen from our construction above, the proper radial-like coordinate for the Einstein-Rosen "bridge" wormhole is η which is related to the Einstein-Rosen coordinate u via *non-smooth* transformation:

$$u = \mathrm{sign}(\eta)\,\sqrt{|\eta|} \quad , \quad \text{i.e. } \; u^2 = |\eta| \; . \tag{69}$$

Thus, we conclude that solution (64) is the proper self-consistent formulation of the original Einstein-Rosen "bridge" wormhole where the presence of *LL-brane* matter source at the "throat" plays crucial role for its well-definiteness.

6 Conclusions

I this work we have continued to explore the use of codimension-one *LL-branes* for construction of wormhole solutions of Einstein equations, in the present case – constructing asymmetric wormholes. We have strongly emphasized the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields:

(i) The *LL-brane* automatically locates itself on (one of) the horizon(s) of the bulk space-time geometry ("horizon straddling");

(ii) The *LL-brane* tension is an additional *dynamical degree of freedom* unlike the case of standard Nambu-Goto p-branes (where it is a given *ad hoc* constant), and which might in particular acquire negative values;

(iii) The *LL-brane* stress-energy tensor provides the appropriate source term on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole solutions;

(iv) Electrically neutral *LL-branes* produce *symmetric* wormholes, *i.e.*, where both left and right "universes" are related via reflection symmetry and where, in particular, the Misner-Wheeler "charge without charge" [21] phenomenon is observed;

(v) The wormhole reflection symmetry is broken through the natural couplings of the *LL-brane* to bulk space-time gauge fields (Maxwell and 3-index Kalb-Ramond). In this way the *LL-brane* dynamically generates non-zero Coulomb field-strength in the "right" universe and non-zero cosmological constant either in the "left" or in the "right" universe which enable the existence of *asymmetric* (with *no* reflection symmetry) wormholes.

We have specifically stressed the crucial role of *LL-branes* already in the case of the "mother of all wormholes" – the classic Einstein-Rosen "bridge" manifold [19].

Finally, let us mention the crucial role of *LL-branes* in constructing non-trivial examples of *non-singular* black holes, *i.e.*, solutions of Einstein equations with black hole type geometry in the bulk space-time, in particular possessing horizons, but with *no* space-time singularities in the center of the geometry. For further details we refer to [28], where a solution of the Einstein-Maxwell-Kalb-Ramond system coupled to a charged *LL-brane* has been obtained describing a regular black hole. The space-time manifold of the latter consists of de Sitter interior region and exterior Reissner-Nordström region glued together along their common horizon (it is the *inner* horizon from the Reissner-Nordström side).

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